

# LATERAL LOADING OF A PILE IN LAYERED SOIL USING THE STRAIN WEDGE MODEL

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**ABSTRACT:** Beam on elastic foundation theory provides an efficient solution for the problem of a laterally loaded pile. The accuracy of such a solution depends upon the characterization of the interaction between the pile and the surrounding soil. A particularly good representation of the soil-pile interaction yields a more realistic solution. While traditional nonlinear "p-y" characterization provides reasonable assessment for a wide range of loaded piles, it has been found that the p-y curve (or the modulus of subgrade reaction) depends on pile properties (width, shape, bending stiffness, and pile-head conditions) as well as soil properties. The strain wedge model allows the assessment of the nonlinear p-y curve response of a laterally loaded pile based on the envisioned relationship between the three-dimensional response of a flexible pile in the soil to its one-dimensional beam on elastic foundation parameters. In addition, the strain wedge model employs stress-strain-strength behavior of the soil as established from the triaxial test and the effective stress condition to evaluate the mobilized soil behavior.

## INTRODUCTION

The strain wedge (SW) model is an approach that has been developed to predict the response of a flexible pile under lateral loading (Norris 1986). The main concept associated with the SW model is that traditional one-dimensional beam on elastic foundation (BEF) pile response parameters can be characterized in terms of three-dimensional soil-pile interaction behavior. The strain wedge model was initially established to analyze a free-head pile embedded in one type of uniform soil (sand or clay). However, the SW model has been improved and modified through additional research to accommodate a laterally loaded pile embedded in multiple soil layers (sand and clay). The strain wedge model has been further modified to include the effect of pile head conditions on soil-pile behavior. The main objective behind the development of the SW model is to solve the BEF problem of a laterally loaded pile based on the envisioned soil-pile interaction and its dependence on both soil and pile properties.

The problem of a laterally loaded pile in layered soil has been solved by Reese (1977) as a BEF based on modeling the soil response by p-y curves. However, as mentioned by Reese (1983), the p-y curve employed does not account for soil continuity and pile properties such as pile stiffness, pile cross-section shape, and pile head conditions. Such is the subject matter of this paper.

## THEORETICAL BASIS OF STRAIN WEDGE MODEL CHARACTERIZATION

The SW model parameters are related to an envisioned three-dimensional passive wedge of soil developing in front of the pile. The basic purpose of the SW model is to relate stress-strain-strength behavior of the soil in the wedge to one-dimensional BEF parameters. The SW model is, therefore, able to provide a theoretical link between the more complex three-dimensional soil-pile interaction and the simpler one-dimensional BEF characterization. The previously noted

correlation between the SW model response and BEF characterization reflects the following interdependence:

- the horizontal soil strain ( $\epsilon$ ) in the developing passive wedge in the front of the pile to the deflection pattern ( $y$  versus depth,  $x$ ) of the pile
- the horizontal soil stress change ( $\Delta\sigma_h$ ) in the developing passive wedge to the soil-pile reaction ( $p$ ) associated with BEF
- the nonlinear variation in the Young's modulus ( $E = \Delta\sigma_h/\epsilon$ ) of the soil to the nonlinear variation in the modulus of soil subgrade reaction ( $E_s = p/y$ ) associated with BEF characterization

These analytical relations reflect soil-pile interaction response characterized by the SW model that will be illustrated later in this paper. The reason for linking the SW model to BEF analysis is to allow the appropriate selection of BEF parameters to solve the following fourth-order ordinary differential equation:

$$EI \left( \frac{d^4 y}{dx^4} \right) + E_s(x) = 0 \quad (1)$$

The closed form solution of (1) has been obtained by Matlock and Reese (1961) for the case of uniform soil. To appreciate the SW model's enhancement of BEF analysis, one should first consider the governing analytical formulations related to the passive wedge in front of the pile, the soil's stress-strain relationship, and the related soil-pile interaction.

## SOIL PASSIVE WEDGE CONFIGURATION IN UNIFORM SOIL

The SW model represents the mobilized passive wedge in front of the pile, which is characterized by base angles,  $\Theta_m$  and  $\beta_m$ ; the current passive wedge depth  $h$ ; and the spread of the wedge fan angle,  $\varphi_m$  (the mobilized friction angle). The horizontal stress change at the passive wedge face,  $\Delta\sigma_h$ , and the side shear,  $\tau$ , act as shown in Fig. 1. One of the main assumptions associated with the SW model is that the deflection pattern of the pile is taken to be linear over the controlling depth of the soil near the pile top, resulting in a linearized deflection angle,  $\delta$ , as seen in Fig. 2. The relationship between the actual (closed form solution) and linearized deflection patterns has been established by Norris (1986). This assumption allows uniform horizontal and vertical soil strains to be assessed (as seen later in Fig. 6). Changes in the shape and depth of the passive wedge, along with changes in the state of load-

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Note. Discussion open until September 1, 1998. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on June 16, 1997. This paper is part of the *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 124, No. 4, April, 1998. ©ASCE, ISSN 1090-0241/98/0004-0303-0315/\$4.00 + \$.50 per page. Paper No. 16004.